

Study of nonleptonic $B_q^* \rightarrow D_q V$ and $P_q D^*$ weak decays

Qin Chang,^{1,2,3} Xiaohui Hu,¹ Junfeng Sun,¹ Xiaolin Wang,¹ and Yueling Yang¹

¹*Institute of Particle and Nuclear Physics,*

Henan Normal University, Xinxiang 453007, China

²*Institute of Particle Physics, Central Normal University, Wuhan 430079, China*

³*State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics,*

Chinese Academy of Sciences, Beijing 100190, China

Abstract

Motivated by the powerful capability of measurement for the b -flavored hadron rare decays at LHC and SuperKEKB/Belle-II, the nonleptonic $\bar{B}^* \rightarrow D\bar{D}^*$, $D\rho^-$, DK^{*-} , πD^* and KD^* weak decays are studied in detail. With the amplitudes calculated with factorization approach and the form factors of B^* transition into pseudoscalar meson evaluated with the BSW model, branching fractions and polarization fractions are firstly presented. Numerically, the CKM-favored $\bar{B}_q^* \rightarrow D_q D_s^{*-}$ and $D_q \rho^-$ decays have large branching fractions, $\sim 10^{-8}$, which should be sought for with priority and firstly observed by LHC and Belle-II experiments. The $\bar{B}_q^* \rightarrow D_q K^*$ and $D_q \rho$ decays are dominated by the longitudinal polarization states. While, the parallel polarization fractions of $\bar{B}_q^* \rightarrow D_q \bar{D}^*$ decays are comparable with the longitudinal ones, numerically, $f_{\parallel} + f_L \simeq 95\%$ and $f_L : f_{\parallel} \simeq 5 : 4$. Some comparisons between $\bar{B}_q^{*0} \rightarrow D_q V$ and their corresponding $\bar{B}_q^0 \rightarrow D_q^* V$ decays are performed, and the relation $f_{L,\parallel}(\bar{B}^{*0} \rightarrow DV) \simeq f_{L,\parallel}(\bar{B}^0 \rightarrow D^{*+} V^-)$ is presented. Besides, with the implication of $SU(3)$ flavor symmetry, some useful ratios R_{du} and R_{ds} are discussed in detail, and suggested to be verified experimentally.

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I. INTRODUCTION

The b physics plays an important role in testing the flavor dynamics of Standard Model (SM), exploring the source of CP violation, searching the indirect hints of new physics, investigating the underlying mechanisms of QCD *et al.*, and thus attracts much experimental and theoretical attention. With the successful performance of BABAR, Belle, CDF and D0 in the past years, many $B_{u,d,s}$ meson decays have been well measured. Thanks to the ongoing LHCb experiment [1] at LHC and forthcoming Belle-II experiment [2] at SuperKEKB, experimental analysis of B meson decays is entering a new frontier of precision. By then, besides $B_{u,d,s}$ mesons, the rare decays of some other b -flavored hadrons are hopefully to be observed, which may provide much more extensive space for b physics.

The excited states $B_{u,d,s}^*$ with quantum number of $n^{2s+1}L_J = 1^3S_1$ and $J^P = 1^-$ (n , L , s , J and P are the quantum numbers of radial, orbital, spin, total angular momenta and parity, respectively), which will be referred as B^* in this paper, had been observed by CLEO, Belle, LHCb and so on [3]. However, except for their masses, there is no more experimental information due to the fact that the production of B^* mesons are mainly through $\Upsilon(5S)$ decays at e^+e^- colliders and the integrated luminosity is not high enough for probing the B^* rare decays. Moreover, B^* decays are dominated by the radiative processes $B^* \rightarrow B\gamma$, and the other decay modes are too rare to be measured easily. Fortunately, with annual integrated luminosity $\sim 13 \text{ ab}^{-1}$ [2] and the cross section of $\Upsilon(5S)$ production in e^+e^- collisions $\sigma(e^+e^- \rightarrow \Upsilon(5S)) = (0.301 \pm 0.002 \pm 0.039) \text{ nb}$ [4], it is expected that about 4×10^9 $\Upsilon(5S)$ samples could be produced per year at the forthcoming super-B factory SuperKEKB/Belle-II, which implies that the B^* rare decays with branching fractions $\gtrsim 10^{-9}$ are possible to be observed. Besides, due to the much larger production cross section of pp collisions, experiments at LHC [5, 6] also possibly provide some experimental information for B^* decays.

With the rapid development of experiment, accordingly, the theoretical evaluations for B^* weak decays are urgently needed and worthwhile. Nonleptonic B^* weak decays allow one to overconstrain parameters obtained from B meson decay, test various models and improve our understanding on the strong interactions and the mechanism responsible for heavy meson weak decay. The observation of an anomalous production rate of B^* weak decays would be a hint of possible new physics beyond SM. In addition, the B^* weak decay provide one unique opportunity of observing the weak decay of a vector meson, where polarization effects can

be used as tests of the underlying structure and dynamics of hadrons. To our knowledge, few previous theoretical works come close to studying B^* weak decays. Compared with the $B^* \rightarrow PP$ decays, which are suppressed dynamically by the orbital angular momentum of final states, $B^* \rightarrow PV$ decays are expected to have much larger branching fractions, and hence generally much easier to be measured. So, in this paper, we will estimate the observables of nonleptonic two-body $B^* \rightarrow PV$ weak decay to offer a ready reference.

Our paper is organized as follows. In section II, after a brief review of the effective Hamiltonian and factorization approach, the explicit amplitudes of $B_{u,d,s}^* \rightarrow D_{u,d,s}^{(*)}M$ decays are calculated. In sections III, the numerical results and discussions are presented. Finally, we summarize in section IV.

II. THEORETICAL FRAMEWORK

Within SM, the effective Hamiltonian responsible for nonleptonic B^* weak decay is [7]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q,q'=u,c} \left[V_{qb}V_{q'p}^* \sum_{i=1}^2 C_i(\mu)O_i(\mu) + V_{qb}V_{qp}^* \sum_{i=3}^{10} C_i(\mu)O_i(\mu) \right] + \text{h.c.}, \quad (1)$$

where $p = d$ or s , $V_{qb}V_{q'p}^*$ is the product of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements; C_i are Wilson coefficients, which describe the short-distance contributions and are calculated perturbatively; The explicit expressions of local four-quark operators O_i are

$$O_1 = (\bar{q}_\alpha b_\alpha)_{V-A} (\bar{p}_\beta q'_\beta)_{V-A}, \quad O_2 = (\bar{q}_\alpha b_\beta)_{V-A} (\bar{p}_\alpha q'_\beta)_{V-A}, \quad (2)$$

$$O_3 = (\bar{p}_\alpha b_\alpha)_{V-A} \sum_{p'} (\bar{p}'_\beta p'_\beta)_{V-A}, \quad O_4 = (\bar{p}_\alpha b_\beta)_{V-A} \sum_{p'} (\bar{p}'_\beta p'_\alpha)_{V-A}, \quad (3)$$

$$O_5 = (\bar{p}_\alpha b_\alpha)_{V-A} \sum_{p'} (\bar{p}'_\beta p'_\beta)_{V+A}, \quad O_6 = (\bar{p}_\alpha b_\beta)_{V-A} \sum_{p'} (\bar{p}'_\beta p'_\alpha)_{V+A}, \quad (4)$$

$$O_7 = (\bar{p}_\alpha b_\alpha)_{V-A} \sum_{p'} \frac{3}{2} Q_{p'} (\bar{p}'_\beta p'_\beta)_{V+A}, \quad O_8 = (\bar{p}_\alpha b_\beta)_{V-A} \sum_{p'} \frac{3}{2} Q_{p'} (\bar{p}'_\beta p'_\alpha)_{V+A}, \quad (5)$$

$$O_9 = (\bar{p}_\alpha b_\alpha)_{V-A} \sum_{p'} \frac{3}{2} Q_{p'} (\bar{p}'_\beta p'_\beta)_{V-A}, \quad O_{10} = (\bar{p}_\alpha b_\beta)_{V-A} \sum_{p'} \frac{3}{2} Q_{p'} (\bar{p}'_\beta p'_\alpha)_{V-A}, \quad (6)$$

where $(\bar{q}_1 q_2)_{V\pm A} = \bar{q}_1 \gamma_\mu (1 \pm \gamma_5) q_2$, α and β are color indices, $Q_{p'}$ is the electric charge of the quark p' in the unit of $|e|$, and p' denotes the active quark at the scale $\mu \sim \mathcal{O}(m_b)$, *i.e.*, $p' = u, d, c, s, b$.

To obtain the decay amplitudes, the remaining and also the most intricate work is how to calculate hadronic matrix elements $\langle PV | O_i | B^* \rangle$. With the factorization approach [8–

11] based on the color transparency mechanism [12, 13], in principle, the hadronic matrix element could be factorized as

$$\langle PV|O_i|B^*\rangle = a \langle P|J_\mu|B^*\rangle \langle V|J^\mu|0\rangle + b \langle V|J_\mu|B^*\rangle \langle P|J^\mu|0\rangle + c \langle PV|J_\mu|0\rangle \langle 0|J^\mu|B^*\rangle. \quad (7)$$

Due to the unnecessary complexity of hadronic matrix element $\langle V|J_\mu|B^*\rangle$ and power suppression of annihilation contributions, we only consider one simple scenario where pseudoscalar meson pick up the spectator quark in B^* meson, *i.e.*, $a = 1$, $b = 0$ and $c = 0$ in Eq.(7) for the moment. Two current matrix elements can be further parameterized by decay constants and transition form factors,

$$\langle V(p, \epsilon)|\bar{q}_1\gamma_\mu q_2|0\rangle = f_V m_V \epsilon_\mu^*, \quad (8)$$

$$\langle P(p_P)|\bar{q}\gamma_\mu b|\bar{B}^*(p_{B^*}, \eta)\rangle = \frac{2V(q^2)}{m_{B^*} + m_P} \varepsilon_{\mu\nu\rho\sigma} \eta^\nu p_P^\rho p_{B^*}^\sigma, \quad (9)$$

$$\begin{aligned} \langle P(p_P)|\bar{q}\gamma^\mu\gamma_5 b|\bar{B}^*(p_{B^*}, \eta)\rangle &= i2m_{B^*} A_0(q^2) \frac{\eta \cdot q}{q^2} q^\mu \\ &+ i(m_P + m_{B^*}) A_1(q^2) \left(\eta^\mu - \frac{\eta \cdot q}{q^2} q^\mu \right) \\ &+ iA_2(q^2) \frac{\eta \cdot q}{m_P + m_{B^*}} \left[(p_{B^*} + p_P)^\mu - \frac{(m_{B^*}^2 - m_P^2)}{q^2} q^\mu \right], \end{aligned} \quad (10)$$

where ϵ and η are the polarization vector, f_V is the decay constant of vector meson, V and $A_{0,1,2}$ are transition form factors, $q = p_{B^*} - p_P$ and the sign convention $\epsilon^{0123} = 1$. Even though some improved approaches, such as the QCD factorization [14, 15], the perturbative QCD scheme [16, 17] and the soft-collinear effective theory [18–21], are presented to evaluate higher order QCD corrections and reduce the renormalization scale dependence, the naive factorization (NF) approximation is a useful tool of theoretical estimation. Because there is no available experimental measurement for now, the NF approach is good enough to give a preliminary analysis, and so adopted in our evaluation.

With the above definitions, the hadronic matrix elements considered here can be decomposed into three scalar invariant amplitudes $S_{1,2,3}$,

$$\langle PV|O_i|B^*\rangle = \epsilon^{*\mu} \eta^\nu \left\{ S_1 g_{\mu\nu} + S_2 \frac{(p_{B^*} + p_P)_\mu p_{V\nu}}{m_{B^*} m_V} + iS_3 \varepsilon_{\mu\nu\rho\sigma} \frac{2p_{B^*}^\rho p_P^\sigma}{m_{B^*} m_V} \right\}, \quad (11)$$

where the amplitudes $S_{1,2,3}$ describe the s , d , p wave contributions, respectively, and are

explicitly written as

$$S_1 = -if_V(m_{B^*} + m_P)m_V A_1, \quad (12)$$

$$S_2 = -i2f_V m_{B^*} m_V^2 \frac{A_2}{m_{B^*} + m_P}, \quad (13)$$

$$S_3 = +i2f_V m_{B^*} m_V^2 \frac{V}{m_{B^*} + m_P}. \quad (14)$$

Alternatively, one can choose the helicity amplitudes H^λ ($\lambda = 0, +, -$),

$$H_{PV}^0 = -S_1 x - S_2(x^2 - 1), \quad (15)$$

$$H_{PV}^\pm = -S_1 \pm S_3 \sqrt{x^2 - 1}, \quad (16)$$

with

$$x \equiv \frac{p_{B^*} p_V}{m_{B^*} m_V} = \frac{m_{B^*}^2 - m_P^2 + m_V^2}{2m_{B^*} m_V}. \quad (17)$$

Now, with the formulae given above and the effective coefficients α_i defined as

$$\alpha_1 = C_1 + \frac{C_2}{N_c}, \quad \alpha_2 = C_2 + \frac{C_1}{N_c}, \quad \alpha_4 = C_4 + \frac{C_3}{N_c}, \quad \alpha_{4,EW} = C_{10} + \frac{C_9}{N_c}, \quad (18)$$

we present the amplitudes of nonleptonic two-body \bar{B}^* decays as follows:

- For $\bar{B}_q^* \rightarrow D_q \bar{D}^*$ decays (the spectator $q = u, d$ and s),

$$\mathcal{A}^\lambda(\bar{B}_q^* \rightarrow D_q D^{*-}) = H_{DD^{*-}}^\lambda \left[V_{cb} V_{cd}^* (\alpha_1 + \alpha_4 + \alpha_{4,EW}) + V_{ub} V_{ud}^* (\alpha_4 + \alpha_{4,EW}) \right], \quad (19)$$

$$\mathcal{A}^\lambda(\bar{B}_q^* \rightarrow D_q D_s^{*-}) = H_{DD_s^{*-}}^\lambda \left[V_{cb} V_{cd}^* (\alpha_1 + \alpha_4 + \alpha_{4,EW}) + V_{ub} V_{ud}^* (\alpha_4 + \alpha_{4,EW}) \right]. \quad (20)$$

- For $\bar{B}_q^{*0} \rightarrow D_q V$ decays (the spectator $q = d$ and s , the $V = \rho^-$ and K^{*-}),

$$\mathcal{A}^\lambda(\bar{B}_q^{*0} \rightarrow D_q \rho^-) = H_{D\rho^-}^\lambda V_{cb} V_{cd}^* \alpha_1, \quad (21)$$

$$\mathcal{A}^\lambda(\bar{B}_q^{*0} \rightarrow D_q K^{*-}) = H_{DK^{*-}}^\lambda V_{cb} V_{cs}^* \alpha_1. \quad (22)$$

- For $\bar{B}^* \rightarrow \pi D^*$ decays,

$$\mathcal{A}^\lambda(B^{*-} \rightarrow \pi^- \bar{D}^{*0}) = H_{\pi^- \bar{D}^{*0}}^\lambda V_{ub} V_{cd}^* \alpha_2, \quad (23)$$

$$\sqrt{2} \mathcal{A}^\lambda(B^{*-} \rightarrow \pi^0 D^{*-}) = H_{\pi^0 D^{*-}}^\lambda V_{ub} V_{cd}^* \alpha_1, \quad (24)$$

$$\sqrt{2} \mathcal{A}^\lambda(B^{*-} \rightarrow \pi^0 D_s^{*-}) = H_{\pi^0 D_s^{*-}}^\lambda V_{ub} V_{cs}^* \alpha_1, \quad (25)$$

$$-\sqrt{2} \mathcal{A}^\lambda(\bar{B}^{*0} \rightarrow \pi^0 D^{*0}) = H_{\pi^0 D^{*0}}^\lambda V_{cb} V_{ud}^* \alpha_2, \quad (26)$$

$$-\sqrt{2} \mathcal{A}^\lambda(\bar{B}^{*0} \rightarrow \pi^0 \bar{D}^{*0}) = H_{\pi^0 \bar{D}^{*0}}^\lambda V_{ub} V_{cd}^* \alpha_2, \quad (27)$$

$$\mathcal{A}^\lambda(\bar{B}^{*0} \rightarrow \pi^+ D^{*-}) = H_{\pi^+ D^{*-}}^\lambda V_{ub} V_{cd}^* \alpha_1, \quad (28)$$

$$\mathcal{A}^\lambda(\bar{B}^{*0} \rightarrow \pi^+ D_s^{*-}) = H_{\pi^+ D_s^{*-}}^\lambda V_{ub} V_{cs}^* \alpha_1. \quad (29)$$

- For $\bar{B}^* \rightarrow KD^*$ decays,

$$\mathcal{A}^\lambda(B^{*-} \rightarrow K^- \bar{D}^{*0}) = H_{K^- \bar{D}^{*0}}^\lambda V_{ub} V_{cs}^* \alpha_2, \quad (30)$$

$$\mathcal{A}^\lambda(\bar{B}^{*0} \rightarrow \bar{K}^0 \bar{D}^{*0}) = H_{\bar{K}^0 \bar{D}^{*0}}^\lambda V_{ub} V_{cs}^* \alpha_2, \quad (31)$$

$$\mathcal{A}^\lambda(\bar{B}^{*0} \rightarrow \bar{K}^0 D^{*0}) = H_{\bar{K}^0 D^{*0}}^\lambda V_{cb} V_{us}^* \alpha_2, \quad (32)$$

$$\mathcal{A}^\lambda(\bar{B}_s^{*0} \rightarrow K^+ D_s^{*-}) = H_{K^+ D_s^{*-}}^\lambda V_{ub} V_{cd}^* \alpha_1, \quad (33)$$

$$\mathcal{A}^\lambda(\bar{B}_s^{*0} \rightarrow K^0 \bar{D}^{*0}) = H_{K^0 \bar{D}^{*0}}^\lambda V_{ub} V_{cd}^* \alpha_2, \quad (34)$$

$$\mathcal{A}^\lambda(\bar{B}_s^{*0} \rightarrow K^0 D^{*0}) = H_{K^0 D^{*0}}^\lambda V_{cb} V_{ud}^* \alpha_2, \quad (35)$$

$$\mathcal{A}^\lambda(\bar{B}_s^{*0} \rightarrow K^+ D_s^{*-}) = H_{K^+ D_s^{*-}}^\lambda V_{ub} V_{cs}^* \alpha_1. \quad (36)$$

In the rest frame of \bar{B}^* meson, the branching fraction can be written as

$$\mathcal{B}(\bar{B}^* \rightarrow PV) = \frac{1}{3} \frac{G_F^2}{2} \frac{1}{8\pi} \frac{p_c}{m_{B^*}^2 \Gamma_{\text{tot}}(B^*)} \sum_\lambda |\mathcal{A}_\lambda(\bar{B}^* \rightarrow PV)|^2, \quad (37)$$

where the momentum of final states is

$$p_c = \frac{\sqrt{[m_{B^*}^2 - (m_P + m_V)^2][m_{B^*}^2 - (m_P - m_V)^2]}}{2m_{B^*}}. \quad (38)$$

The longitudinal, parallel and perpendicular polarization fractions are defined as

$$f_{L,\parallel,\perp} = \frac{|\mathcal{A}_{0,\parallel,\perp}|^2}{|\mathcal{A}_0|^2 + |\mathcal{A}_\parallel|^2 + |\mathcal{A}_\perp|^2}, \quad (39)$$

where \mathcal{A}_\parallel and \mathcal{A}_\perp are parallel and perpendicular amplitudes gotten through

$$\mathcal{A}_{\parallel,\perp} = \frac{1}{\sqrt{2}}(\mathcal{A}_- \pm \mathcal{A}_+) \quad (40)$$

for \bar{B}^* decays.

III. NUMERICAL RESULTS AND DISCUSSION

Firstly, we would like to clarify the input parameters used in our numerical evaluations. For the CKM matrix elements, we adopt the Wolfenstein parameterization [22] and choose the four parameters A , λ , ρ and η as [23]

$$A = 0.810_{-0.024}^{+0.018}, \quad \lambda = 0.22548_{-0.00034}^{+0.00068}, \quad \bar{\rho} = 0.1453_{-0.0073}^{+0.0133}, \quad \bar{\eta} = 0.343_{-0.012}^{+0.011}, \quad (41)$$

with $\bar{\rho} = \rho(1 - \frac{\lambda^2}{2})$ and $\bar{\eta} = \eta(1 - \frac{\lambda^2}{2})$.

The decay constants of light vector mesons are [24]

$$f_\rho = (216 \pm 3) \text{ MeV}, \quad f_{K^*} = (220 \pm 5) \text{ MeV}. \quad (42)$$

For the decay constants of $D_{(s)}^*$ mesons, we will take [25]

$$f_{D^*} = (252.2 \pm 22.3 \pm 4) \text{ MeV}, \quad f_{D_s^*} = (305.5 \pm 26.8 \pm 5) \text{ MeV}, \quad (43)$$

which agree well with the results of the other QCD sum rules [26, 27] and lattice QCD with $N_f = 2$ [28].

TABLE I: The numerical results of form factors within BSW model.

Transition	$V(0)$	$A_1(0)$	$A_2(0)$
$B^* \rightarrow D$	0.76	0.75	0.62
$B^* \rightarrow K$	0.41	0.42	0.35
$B^* \rightarrow \pi$	0.35	0.38	0.30
$B_s^* \rightarrow D_s$	0.72	0.69	0.59
$B_s^* \rightarrow K$	0.30	0.29	0.26

Besides the decay constants, the $B^* \rightarrow P$ transition form factors are also essential inputs to estimate branching ratios for nonleptonic $B^* \rightarrow PV$ decay. In this paper, the Bauer-Stech-Wirbel (BSW) model [10] is employed to evaluate the form factors $A_1(0)$, $A_2(0)$ and $V(0)$, which could be written as the overlap integrals of wave functions of mesons [10],

$$V^{B^* \rightarrow P}(0) = \frac{m_b - m_q}{m_{B^*} - m_P} J^{B^* \rightarrow P}, \quad (44)$$

$$A_1^{B^* \rightarrow P}(0) = \frac{m_b + m_q}{m_{B^*} + m_P} J^{B^* \rightarrow P}, \quad (45)$$

$$A_2^{B^* \rightarrow P}(0) = \frac{2m_{B^*}}{m_{B^*} - m_P} A_0^{B^* \rightarrow P}(0) - \frac{m_{B^*} + m_P}{m_{B^*} - m_P} A_1^{B^* \rightarrow P}(0), \quad (46)$$

$$A_0^{B^* \rightarrow P}(0) = \int d^2 p_\perp \int_0^1 dx \varphi_P(\vec{p}_\perp, x) \sigma_z \varphi_V^{1,0}(\vec{p}_\perp, x), \quad (47)$$

$$J^{B^* \rightarrow P} = \sqrt{2} \int d^2 p_\perp \int_0^1 dx \varphi_P(\vec{p}_\perp, x) i\sigma_y \varphi_V^{1,-1}(\vec{p}_\perp, x), \quad (48)$$

where \vec{p}_\perp is the transverse quark momentum, $\sigma_{y,z}$ are the Pauli matrix acting on the spin indices of the decaying quark, and m_q represents the mass of nonspectator quark of pseudoscalar meson. With the meson wave function $\varphi_M(\vec{p}_\perp, x)$ as solution of a relativistic scalar

harmonic oscillator potential [10], and $\omega = 0.4$ GeV which determines the average transverse quark momentum through $\langle p_\perp^2 \rangle = \omega^2$, we get the numerical results of the transition form factors summarized in Table I. In our following evaluation, these numbers and 15% of them are used as default inputs and uncertainties, respectively.

To evaluate the branching fractions, the total decay widths (or lifetimes) $\Gamma_{\text{tot}}(B^*)$ are necessary. However, there is no available experimental or theoretical information for $\Gamma_{\text{tot}}(B^*)$ until now. Because of the fact that the QED radiative processes $B^* \rightarrow B\gamma$ dominate the decays of B^* mesons, we will take the approximation $\Gamma_{\text{tot}}(B^*) \simeq \Gamma(B^* \rightarrow B\gamma)$. The theoretical predictions on $\Gamma(B^* \rightarrow B\gamma)$ have been widely evaluated in various theoretical models, such as relativistic quark model [29, 30], QCD sum rules [31], light cone QCD sum rules [32], light front quark model [33], heavy quark effective theory with vector meson dominance hypothesis [34] or covariant model [35]. In this paper, the most recent results [33, 35]

$$\Gamma(B^{*+} \rightarrow B^+ \gamma) = (468_{-75}^{+73}) \text{ eV}, \quad (49)$$

$$\Gamma(B^{*0} \rightarrow B^0 \gamma) = (148 \pm 20) \text{ eV}, \quad (50)$$

$$\Gamma(B_s^{*0} \rightarrow B_s^0 \gamma) = (68 \pm 17) \text{ eV}, \quad (51)$$

which agree with the other theoretical results, are approximately treated as Γ_{tot} in our numerical estimate.

With the aforementioned values of input parameters and the theoretical formula, we present theoretical predictions for the observables of $\bar{B}^* \rightarrow D\bar{D}^*$, $D\rho$, DK^* , πD^* , KD^* decays, in which only the (color-suppressed) tree induced decay modes are evaluated due to that the branching fractions of loop induced decays are very small and hardly to be measured soon. Our numerical results for the branching fractions and the polarization fractions are summarized in Tables II and III. In Table II, the first, second and third theoretical errors are caused by uncertainties of the CKM parameters, hadronic parameters (decay constants and form factors) and total decay widths, respectively. From Tables II and III, it could be found that:

- (1) The hierarchy of branching fractions is clear. (i) The branching fractions of $\bar{B}^* \rightarrow \pi D^*$ and KD^* decays are much smaller than the ones of $\bar{B}^* \rightarrow D\bar{D}^*$, $D\rho$ and DK^* decays, which is caused by that the form factors of $\bar{B}^* \rightarrow D$ transition are much larger than those of $\bar{B}^* \rightarrow \pi$ and $\bar{B}^* \rightarrow K$ transitions. (ii) For $\bar{B}^* \rightarrow D\bar{D}^*$, $D\rho$ and DK^* decays,

TABLE II: The CP -averaged branching fractions of nonleptonic B^* weak decays.

Decay modes	Class	CKM factors	\mathcal{B}
$B^{*-} \rightarrow D^0 D^{*-}$	T, P, P_{ew}	λ^3	$(3.9^{+0.2+1.3+0.7}_{-0.2-1.1-0.5}) \times 10^{-10}$
$\bar{B}^{*0} \rightarrow D^+ D^{*-}$	T, P, P_{ew}	λ^3	$(1.2^{+0.1+0.4+0.2}_{-0.1-0.4-0.1}) \times 10^{-9}$
$B^{*-} \rightarrow D^0 D_s^{*-}$	T, P, P_{ew}	λ^2	$(1.1^{+0.1+0.4+0.2}_{-0.1-0.3-0.1}) \times 10^{-8}$
$\bar{B}^{*0} \rightarrow D^+ D_s^{*-}$	T, P, P_{ew}	λ^2	$(3.4^{+0.2+1.1+0.5}_{-0.2-1.0-0.4}) \times 10^{-8}$
$\bar{B}_s^{*0} \rightarrow D_s^+ D^{*-}$	T, P, P_{ew}	λ^3	$(2.3^{+0.1+0.8+0.8}_{-0.1-0.7-0.5}) \times 10^{-9}$
$\bar{B}_s^{*0} \rightarrow D_s^+ D_s^{*-}$	T, P, P_{ew}	λ^2	$(6.4^{+0.3+2.1+2.1}_{-0.4-1.9-1.3}) \times 10^{-8}$
$\bar{B}^{*0} \rightarrow D^+ K^{*-}$	T	λ^3	$(7.6^{+0.4+1.9+1.2}_{-0.4-1.7-0.9}) \times 10^{-10}$
$\bar{B}_s^{*0} \rightarrow D_s^+ K^{*-}$	T	λ^3	$(1.5^{+0.1+0.4+0.5}_{-0.1-0.3-0.3}) \times 10^{-9}$
$\bar{B}^{*0} \rightarrow D^+ \rho^-$	T	λ^2	$(1.3^{+0.1+0.3+0.2}_{-0.1-0.3-0.2}) \times 10^{-8}$
$\bar{B}_s^{*0} \rightarrow D_s^+ \rho^-$	T	λ^2	$(2.6^{+0.1+0.6+0.9}_{-0.1-0.6-0.5}) \times 10^{-8}$
$B^{*-} \rightarrow \pi^- \bar{D}^{*0}$	C	λ^4	$(3.1^{+0.2+0.8+0.6}_{-0.2-0.6-0.4}) \times 10^{-14}$
$B^{*-} \rightarrow \pi^0 D^{*-}$	T	λ^4	$(4.6^{+0.4+1.4+0.9}_{-0.4-1.2-0.6}) \times 10^{-13}$
$\bar{B}^{*0} \rightarrow \pi^+ D^{*-}$	T	λ^4	$(2.9^{+0.2+0.9+0.5}_{-0.2-0.8-0.3}) \times 10^{-12}$
$\bar{B}^{*0} \rightarrow \pi^0 D^{*0}$	C	λ^2	$(1.2^{+0.1+0.4+0.2}_{-0.1-0.3-0.1}) \times 10^{-10}$
$\bar{B}^{*0} \rightarrow \pi^0 \bar{D}^{*0}$	C	λ^4	$(4.9^{+0.3+1.4+0.8}_{-0.3-1.2-0.6}) \times 10^{-14}$
$B^{*-} \rightarrow \pi^0 D_s^{*-}$	T	λ^3	$(1.3^{+0.1+0.4+0.2}_{-0.1-0.3-0.2}) \times 10^{-11}$
$\bar{B}^{*0} \rightarrow \pi^+ D_s^{*-}$	T	λ^3	$(8.1^{+0.6+2.5+1.3}_{-0.7-2.2-1.0}) \times 10^{-11}$
$B^{*-} \rightarrow K^- \bar{D}^{*0}$	C	λ^3	$(7.4^{+0.6+2.1+1.4}_{-0.6-1.9-1.0}) \times 10^{-13}$
$\bar{B}^{*0} \rightarrow \bar{K}^0 D^{*0}$	C	λ^3	$(1.7^{+0.1+0.5+0.3}_{-0.1-0.4-0.2}) \times 10^{-11}$
$\bar{B}^{*0} \rightarrow \bar{K}^0 \bar{D}^{*0}$	C	λ^3	$(2.3^{+0.2+0.7+0.4}_{-0.2-0.6-0.3}) \times 10^{-12}$
$\bar{B}_s^{*0} \rightarrow K^+ D^{*-}$	T	λ^4	$(4.3^{+0.3+1.2+1.4}_{-0.4-1.1-0.9}) \times 10^{-12}$
$\bar{B}_s^{*0} \rightarrow K^0 D^{*0}$	C	λ^2	$(3.6^{+0.2+1.0+1.2}_{-0.2-0.9-0.7}) \times 10^{-10}$
$\bar{B}_s^{*0} \rightarrow K^0 \bar{D}^{*0}$	C	λ^4	$(1.4^{+0.1+0.4+0.5}_{-0.1-0.3-0.3}) \times 10^{-13}$
$\bar{B}_s^{*0} \rightarrow K^+ D_s^{*-}$	T	λ^3	$(1.2^{+0.1+0.3+0.4}_{-0.1-0.3-0.2}) \times 10^{-10}$

the hierarchy are induced by two factors: one is the CKM factor (see the third column of Table II), the other is $\Gamma_{\text{tot}}(B^{*\pm}) > \Gamma_{\text{tot}}(B_d^{*0}) > \Gamma_{\text{tot}}(B_s^{*0})$ [see Eqs.(49,50,51)].

(2) Besides small form factors, the $\bar{B}^* \rightarrow \pi D^*, KD^*$ decays are either color suppressed or

TABLE III: The polarization fractions f_L and f_{\parallel} (in the units of percent).

Decay modes	f_L	f_{\parallel}
$B^{*-} \rightarrow D^0 D^{*-}$	54_{-2}^{+2}	40_{-2}^{+2}
$\bar{B}^{*0} \rightarrow D^+ D^{*-}$	54_{-2}^{+2}	40_{-2}^{+2}
$B^{*-} \rightarrow D^0 D_s^{*-}$	52_{-2}^{+1}	43_{-2}^{+2}
$\bar{B}^{*0} \rightarrow D^+ D_s^{*-}$	52_{-2}^{+1}	43_{-2}^{+2}
$\bar{B}_s^{*0} \rightarrow D_s^+ D^{*-}$	54_{-2}^{+2}	40_{-2}^{+2}
$\bar{B}_s^{*0} \rightarrow D_s^+ D_s^{*-}$	52_{-2}^{+2}	42_{-2}^{+2}
$\bar{B}^{*0} \rightarrow D^+ K^{*-}$	85_{-1}^{+1}	13_{-1}^{+1}
$\bar{B}_s^{*0} \rightarrow D_s^+ K^{*-}$	85_{-1}^{+1}	13_{-1}^{+1}
$\bar{B}^{*0} \rightarrow D^+ \rho^-$	88_{-1}^{+1}	10_{-1}^{+1}
$\bar{B}_s^{*0} \rightarrow D_s^+ \rho^-$	88_{-1}^{+1}	10_{-1}^{+1}

the CKM factors suppressed, hence have very small branching fractions (see TableII) to be hardly measured soon. Most of the CKM favored and tree-dominated $\bar{B}^* \rightarrow D\bar{D}^*, D\rho, DK^*$ decays, enhanced by the relatively large $\bar{B}^* \rightarrow D$ transition form factors, have large branching fractions, $\gtrsim 10^{-9}$, and thus could be measured in the near future. In particular, branching ratios for $\bar{B}_q^* \rightarrow D_q \bar{D}_s^{*-}, D_q \rho$ decays can reach up to 10^{-8} , and hence should be sought for with priority and firstly observed at the high statistics LHC and Belle-II experiments.

The numerical results and above analyses are based on the NF, in which the QCD corrections are not included. Fortunately, for the color-allowed tree amplitude α_1 , the NF estimate is stable due to the relatively small QCD corrections [15]. For instance, in $B \rightarrow \pi\pi$ and $B \rightarrow D^* L$ decays, the results $\alpha_1(\pi\pi) = (1.020)_{LO} + (0.018 + 0.018i)_{NLO}$ [14] and $\alpha_1(D^* L) = (1.025)_{LO} + (0.019 + 0.013i)_{NLO}$ [15] indicate clearly that the $\mathcal{O}(\alpha_s)$ correction is only about 2% and thus trivial numerically. For the color-suppressed decay modes listed in Tables II, even though the NF estimates would suffer significant $\mathcal{O}(\alpha_s)$ correction (about 46% in $B \rightarrow \pi\pi$ decays for instance [36]), they still escape the experimental scope due to their small branching fractions $< 10^{-9}$, and thus will not be discussed further. In the following analyses, we will pay our attention

only to the color allowed tree-dominated $\bar{B}^* \rightarrow D\bar{D}^*$, $D\rho$, DK^* decays.

- (3) For the $B^{*-} \rightarrow D^0 D_{(s)}^{*-}$ and $\bar{B}^{*0} \rightarrow D^+ D_{(s)}^{*-}$ decays, the $SU(3)$ flavor symmetry implies the relations

$$\mathcal{A}(B^{*-} \rightarrow D^0 D_s^{*-}) \simeq \mathcal{A}(\bar{B}^{*0} \rightarrow D^+ D_s^{*-}), \quad (52)$$

$$\mathcal{A}(B^{*-} \rightarrow D^0 D^{*-}) \simeq \mathcal{A}(\bar{B}^{*0} \rightarrow D^+ D^{*-}). \quad (53)$$

Further considering the theoretical prediction $\Gamma(B^{*+} \rightarrow B^+ \gamma) / \Gamma(B^{*0} \rightarrow B^0 \gamma) \approx 3$ [see Eqs.(49,50)] and assumption $\Gamma_{\text{tot}}(B^*) \simeq \Gamma(B^* \rightarrow B \gamma)$, one may find the ratio

$$R_{\text{du}} \equiv \frac{\mathcal{B}(\bar{B}^{*0} \rightarrow D^+ D_s^{*-})}{\mathcal{B}(B^{*-} \rightarrow D^0 D_s^{*-})} \simeq \frac{\Gamma(B^{*+} \rightarrow B^+ \gamma)}{\Gamma(B^{*0} \rightarrow B^0 \gamma)} \stackrel{\text{theo.}}{\approx} 3, \quad (54)$$

$$R'_{\text{du}} \equiv \frac{\mathcal{B}(\bar{B}^{*0} \rightarrow D^+ D^{*-})}{\mathcal{B}(B^{*-} \rightarrow D^0 D^{*-})} \simeq R_{\text{du}}, \quad (55)$$

which are satisfied in our numerical evaluations. Experimentally, the first relation Eq.(54) is hopeful to be tested soon due to the large branching fractions. For the other potentially detectable $\bar{B}_{d,s}^{*0} \rightarrow D\bar{D}^*$, $D\rho$ and DK^* decay modes, which branching fractions $\gtrsim 10^{-9}$, the U-spin symmetry implies relations

$$\mathcal{A}(\bar{B}^{*0} \rightarrow D^+ D^{*-}) \simeq \mathcal{A}(\bar{B}_s^{*0} \rightarrow D_s^+ D^{*-}), \quad (56)$$

$$\mathcal{A}(\bar{B}^{*0} \rightarrow D^+ D_s^{*-}) \simeq \mathcal{A}(\bar{B}_s^{*0} \rightarrow D_s^+ D_s^{*-}), \quad (57)$$

$$\mathcal{A}(\bar{B}^{*0} \rightarrow D^+ K^{*-}) \simeq \mathcal{A}(\bar{B}_s^{*0} \rightarrow D_s^+ K^{*-}), \quad (58)$$

$$\mathcal{A}(\bar{B}^{*0} \rightarrow D^+ \rho^-) \simeq \mathcal{A}(\bar{B}_s^{*0} \rightarrow D_s^+ \rho^-). \quad (59)$$

As similar to R_{du} , one also could get the ratio and relation

$$R_{\text{ds}} \equiv \frac{\mathcal{B}(\bar{B}^{*0} \rightarrow D^+ D^{*-}, D^+ D_s^{*-}, D^+ K^{*-}, D^+ \rho^-)}{\mathcal{B}(\bar{B}_s^{*0} \rightarrow D_s^+ D^{*-}, D_s^+ D_s^{*-}, D_s^+ K^{*-}, D_s^+ \rho^-)} \simeq \frac{\Gamma(B_s^{*0} \rightarrow B_s^0 \gamma)}{\Gamma(B^{*0} \rightarrow B^0 \gamma)} \stackrel{\text{theo.}}{\approx} 2, \quad (60)$$

which is also satisfied in our numerical evaluation. So, it is obvious that such ratios R_{du} and R_{ds} are useful for probing $\tau_{B^{*0}} / \tau_{B^{*\pm}}$ and $\tau_{B^{*0}} / \tau_{B_s^{*0}}$, respectively, and further testing the theoretical predictions of $\Gamma(B^{*+} \rightarrow B^+ \gamma) / \Gamma(B^{*0} \rightarrow B^0 \gamma)$ and $\Gamma(B_s^{*0} \rightarrow B_s^0 \gamma) / \Gamma(B^{*0} \rightarrow B^0 \gamma)$ in various models, such as the results in Refs. [29–35].

- (4) Besides of branching fraction, the polarization fractions $f_{L,\parallel,\perp}$ are also important observables. For the potentially detectable decay modes with branching fractions $\gtrsim 10^{-9}$,

our numerical results of $f_{L,\parallel}$ are summarized in Table III. For the helicity amplitudes \mathcal{A}_λ , the formal hierarchy pattern

$$\mathcal{A}_0 : \mathcal{A}_- : \mathcal{A}_+ = 1 : \frac{\Lambda_{\text{QCD}}}{m_b} : \left(\frac{\Lambda_{\text{QCD}}}{m_b} \right)^2 \quad (61)$$

is naively expected. Hence, $\bar{B}^* \rightarrow PV$ decays are generally dominated by the longitudinal polarization state and satisfy $f_L \sim 1 - 1/m_{B^*}^2$ [37]. For $\bar{B}^* \rightarrow DV$ ($V = K^*, \rho$) decays, in the heavy-quark limit, the helicity amplitudes H^λ given by Eqs. (15) and (16) could be simplified as

$$\begin{aligned} H_{PV}^0 &\simeq if_V \left[\frac{(m_{B^*} - m_D)(m_{B^*} + m_D)^2}{2m_{B^*}} A_1 + \frac{(m_{B^*} + m_D)(m_{B^*} - m_D)^2}{2m_{B^*}} A_2 \right], \quad (62) \\ H_{PV}^\pm &\simeq if_V \left[\frac{(m_{B^*} - m_D)(m_{B^*} + m_D)^2}{2m_{B^*}} A_1 \mp \frac{(m_{B^*} + m_D)(m_{B^*} - m_D)^2}{2m_{B^*}} V \right] \\ &\quad \cdot \frac{2m_{B^*}m_V}{m_{B^*}^2 - m_D^2}. \end{aligned} \quad (63)$$

The transversity amplitudes could be gotten easily through Eq. (40). Obviously, due to the helicity suppression factor $2m_{B^*}m_V/(m_{B^*}^2 - m_D^2) \sim 2m_V/m_{B^*} \sim \Lambda_{\text{QCD}}/m_b$, the relation of Eq.(61) are roughly fulfilled. As a result, the longitudinal polarization fractions of $\bar{B}^* \rightarrow DK^*$ and $D\rho$ decays are very large (see Table III for numerical results).

It should be noted that above analyses and Eqs. (62) and (63) are based on the case of $m_V^2 \ll m_{B^*}^2$, and thus possibly no longer satisfied by $\bar{B}^* \rightarrow D\bar{D}^*$ decays because of the un-negligible vector mass m_{D^*} . In fact, for the $\bar{B}^* \rightarrow D\bar{D}^*$ decays, Eqs. (15) and (16) are simplified as

$$H_{PV}^0 \simeq if_{D^*} \left[\frac{(m_{B^*} + m_D)m_{B^*}}{2} A_1 + \frac{m_{B^*}}{2(m_{B^*} + m_D)} (m_{B^*}^2 - 4m_{D^*}^2) A_2 \right], \quad (64)$$

$$H_{PV}^\pm \simeq if_{D^*} \left[\frac{(m_{B^*} + m_D)m_{B^*}}{2} A_1 \mp \frac{m_{B^*}}{2(m_{B^*} + m_D)} m_{B^*} \sqrt{m_{B^*}^2 - 4m_{D^*}^2} V \right] \cdot \frac{2m_{D^*}}{m_{B^*}} \quad (65)$$

in which, due to $(m_{D^*}^2 - m_D^2) \ll m_{B^*}^2$, the approximation $x = \frac{m_{B^*}^2 - m_D^2 + m_{D^*}^2}{2m_{B^*}m_{D^*}} \simeq \frac{m_{B^*}}{2m_{D^*}}$ is used. Because the so-called helicity suppression factor $2m_{D^*}/m_{B^*} \sim 0.8$ is not small, which is different from the case of $\bar{B}^* \rightarrow DV$ decays, it could be easily found that the relation of Eq.(61) doesn't follow. Further considering that H_{PV}^\pm are dominated by the term of A_1 in Eq. (65) due to its large coefficient, the relation $f_L(D\bar{D}^*) \sim f_\parallel(D\bar{D}^*) \gg f_\perp(D\bar{D}^*)$ could be easily gotten. Above analyses and findings are confirmed by our numerical results in Table III, which will be tested by future experiments.

- (5) As known, there are many interesting phenomena in B meson decays, so it is worthy to explore the possible correlation between B and B^* decays. Taking $\bar{B}^{*0} \rightarrow D^+\rho^-$ and $\bar{B}^0 \rightarrow D^{*+}\rho^-$ decays as example, we find that the expressions of their helicity amplitudes (the former one have be given by Eqs. (62) and (63)) are similar with each other except for the replacements $\bar{B}^* \leftrightarrow \bar{B}$ and $D \leftrightarrow D^*$ everywhere in Eqs. (62) and (63). As a result, our analyses in item (4) are roughly suitable for $\bar{B}^0 \rightarrow D^{*+}\rho^-$ decay, and the relation

$$f_{L,\parallel}(\bar{B}^{*0} \rightarrow D^+\rho^-) \simeq f_{L,\parallel}(\bar{B}^0 \rightarrow D^{*+}\rho^-) \quad (66)$$

is generally expected. Interestingly, our prediction $f_L^{NF}(\bar{B}^{*0} \rightarrow D^+\rho^-) = (88 \pm 1)\%$ is consistent with the result $f_L^{\text{WSB}}(\bar{B}^0 \rightarrow D^{*+}\rho^-) = 87\%$ [38], which is in a good agreement with the experimental data $f_L^{\text{exp.}}(\bar{B}^0 \rightarrow D^{*+}\rho^-) = (88.5 \pm 1.6 \pm 1.2)\%$ [39]. The relation Eq. (66) follows. In addition, the similar correlation as Eq. (66) also exists in the other B^* and corresponding B decays.

IV. SUMMARY

In this paper, motivated by the experiments of heavy flavor physics at the running LHC and forthcoming SuperKEKB/Belle-II, the nonleptonic $\bar{B}^* \rightarrow D\bar{D}^*$, $D\rho$, DK^* , πD^* , KD^* weak decay modes are evaluated with factorization approach, in which the transition form factors are calculated with the BSW model and the approximation $\Gamma_{\text{tot}}(B^*) \simeq \Gamma(B^* \rightarrow B\gamma)$ is used to evaluate the branching fractions. It is found that: (i) there are some obvious hierarchy among branching fractions, in which the $\bar{B}_q^* \rightarrow D_q\bar{D}_s^{*-}$ and $D_q\rho^-$ decays have large branching fractions $\sim 10^{-8}$, and hence should be sought for with priority at LHC and Belle-II experiments. (ii) With the implication of $SU(3)$ (or U-spin) flavor symmetry, some useful ratios, R_{du} and R_{ds} , are suggested to be verified experimentally. (iii) The $\bar{B}^{*0} \rightarrow DK^*$ and $D\rho$ decays are dominated by the longitudinal polarization states, numerically $f_L \sim [80\%, 90\%]$. While, the parallel polarization fractions of $\bar{B}^* \rightarrow D\bar{D}^*$ decays are comparable with the longitudinal ones, numerically, $f_L : f_{\parallel} \simeq 5 : 4$. In addition, comparing with $B \rightarrow VV$ decays, the relation $f_{L,\parallel}(\bar{B}^{*0} \rightarrow DV) \simeq f_{L,\parallel}(\bar{B}^0 \rightarrow DV)$ is generally expected. These results and findings are waiting for confirmation from future LHC and Belle-II experiments.

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